21 Problems: Proving Trigonometric Identities

Prove each of the following identities.

1. $\sin x \cot x = \cos x$
2. $\tan x + \cos x = \sin x(\sec x + \cot x)$
3. $\dfrac{\cos^4 x - \sin^4 x}{\cos^2 x} = 1 - \tan^2 x$
4. $1 + \cos x = \dfrac{\sin^2 x}{1 - \cos x}$
5. $\tan x + \cot x = \sec x \csc x$
6. $\sec x \cot x \sin x = 1$
7. $\dfrac{\tan x}{\sec x} = \sin x$
8. $\tan x(\cos x + \cot x) = \sin x + 1$
9. $(\sin x - \cos x)^2 = 1 - \sin 2x$
10. $(1 - \sin x)(1 + \sin x) = \cos^2 x$
11. $\dfrac{1}{1 + \cos x} + \dfrac{1}{1 - \cos x} = 2\csc^2 x$
12. $\csc x - \sin x = \cot x \cos x$
13. $\tan x - \cot x = \dfrac{\sin^2 x - \cos^2 x}{\sin x \cos x}$
14. $\cos^4 x - \sin^4 x = \cos 2x$
15. $\sin^2 \dfrac{x}{2} = \dfrac{\tan x + \sin x}{2 \tan x}$
16. $\dfrac{\csc x - 1}{\csc x + 1} = \dfrac{1 - \sin x}{1 + \sin x}$
17. $1 - \sin x = \dfrac{\cos^2 x}{1 + \sin x}$
18. $\cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$
19. $\tan x \sin x + \cos x = \sec x$
20. $\dfrac{1 - \tan^3 x}{1 - \tan x} = \sec^2 x + \tan x$
21. $\dfrac{\sin x}{1 + \cos x} + \dfrac{1 + \cos x}{\sin x} = 2\csc x$

Tips for Proving Trigonometric Identities

1. It is usually best to work on the more complicated side first
2. Look for trigonometric substitutions involving the basic identities (below) that may help simplify things
3. Look for algebraic operations, such as adding fractions, the distributive property, or factoring, that may simplify the side you are working with.
4. If you cannot think of anything else to do, change everything to sines and cosines and see if that helps.
5. Always keep an eye on the side you are not working with to be sure you are working toward it. There is certain sense of direction that accompanies a succesful proof.

Basic Identities

Recriprocal Identities

$$\csc x = \frac{1}{\sin x}$$
$$\sec x = \frac{1}{\cos x}$$
$$\cot x = \frac{1}{\tan x}$$

Ratio Identities

$$\tan x = \frac{\sin x}{\cos x}$$
$$\cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identites

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

Angle Sum & Difference Identites

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

Double Angle Identites

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\cos 2x = 1 - 2\sin^2 x$$
$$\cos 2x = 2\cos^2 x - 1$$

Half Angle Identites

$$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$$
$$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$$