Example from §1.8

1. Let $f(x) = 4x^2 - 8x - 45$.
   (a) Rewrite this quadratic in standard form by completing the square.
   (b) Find the vertex.
   (c) Find all intercepts.
   (d) Find the maximum or minimum value of the function.
   (e) Sketch its graph labeling the vertex and intercepts.

Solution.

(a) *Rewrite this quadratic in standard form by completing the square.*

Understand each of the following steps in completing the square!

$$\begin{aligned} y &= 4x^2 - 8x - 45 \\ &= (4x^2 - 8x) - 45 \\ &= 4(x^2 - 2x) - 45 \\ &= 4(x^2 - 2x + \underline{\quad\quad} - \underline{\quad\quad}) - 45 \\ &= 4(x^2 - 2x + 1 - 1) - 45 \\ &= 4(x-1)^2 - 4 - 45 \\ &= 4(x-1)^2 - 49 \end{aligned}$$

(b) *Find the vertex.*

From standard form, the vertex is $(h, k) = (1, -49)$.

(c) *Find all intercepts.*

For the y-intercept, plug-in $x = 0$. It is $f(0) = -45$.

For the x-intercepts, plugin $y = 0$ and solve:

$$4x^2 - 8x - 45 = 0.$$

Using the quadratic formula, $a = 4, b = -8, c = -45$:

$$\begin{aligned} x &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(-45)}}{2(4)} \\ &= \frac{8 \pm \sqrt{784}}{8} \\ &= \frac{8 \pm 28}{8} \\ &= \frac{8+28}{8}, \frac{8-28}{8} \\ &= \frac{9}{2}, -\frac{5}{2} \end{aligned}$$

(d) *Find the maximum or minimum value of the function.*

Since a is positive, the parabola faces up. So the vertex is a minimum. So the minimum value is -49 which occurs when $x = 1$.

(e) *Sketch its graph labeling the vertex and intercepts.*

Plot the vertex and intercepts and connect the dots with a parabola.

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2. Repeat parts (a) – (e) for $f(x) = -2x^2 - 6x - 2$.